Section 7
Analysis Techniques for Vibratory Data

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Outline and References

Outline:

- Overview
- Time Domain Analysis
- Frequency Domain Analysis
- Summary

PIMS Internet References:

http://www.grc.nasa.gov/Other_Groups/MMAP/PIMS/meit3/

- Details for Power Spectral Density (PSD) and Parseval's Theorem
- Electronic version of this presentation

Overview

Objectives:

- characterize significant traits of the measured data
  - *qualify* – identify significant features or parameters
  - *quantify* – enumerate some aspect
- compare measured data to history, requirements, or predictions
- summarize measured data

Motivations:

- assist investigators and update/maintain knowledge base
- provide feedback to those interested in a data set's relativity
- manage large volume of data

Approaches:

- time domain analysis
- frequency domain analysis

Time Domain Analysis

Objectives:

- isolate acceleration events with respect to time
- threshold acceleration data for limit checking
- correlate acceleration data with other information

Approaches:

- acceleration vs. time → interval processing

- interval average (AVG) acceleration vs. time
- interval root-mean-square (RMS) acceleration vs. time
- interval minimum/maximum (MIN/MAX) acceleration vs. time

Acceleration vs. Time

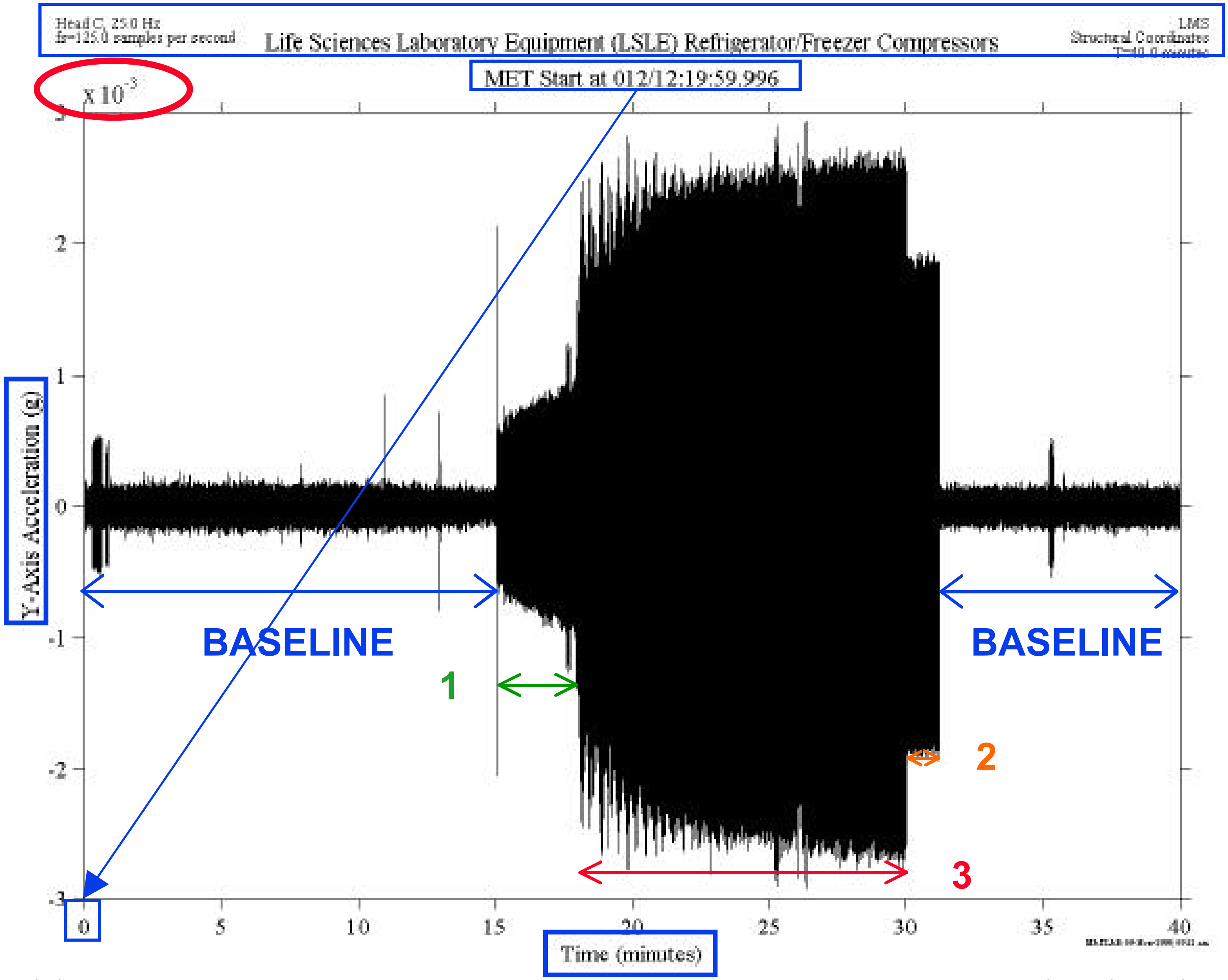
Advantages:

- most precise accounting of the measured data with respect to time
- fundamental approach to quantifying acceleration environment
- "purest" form of the data collected

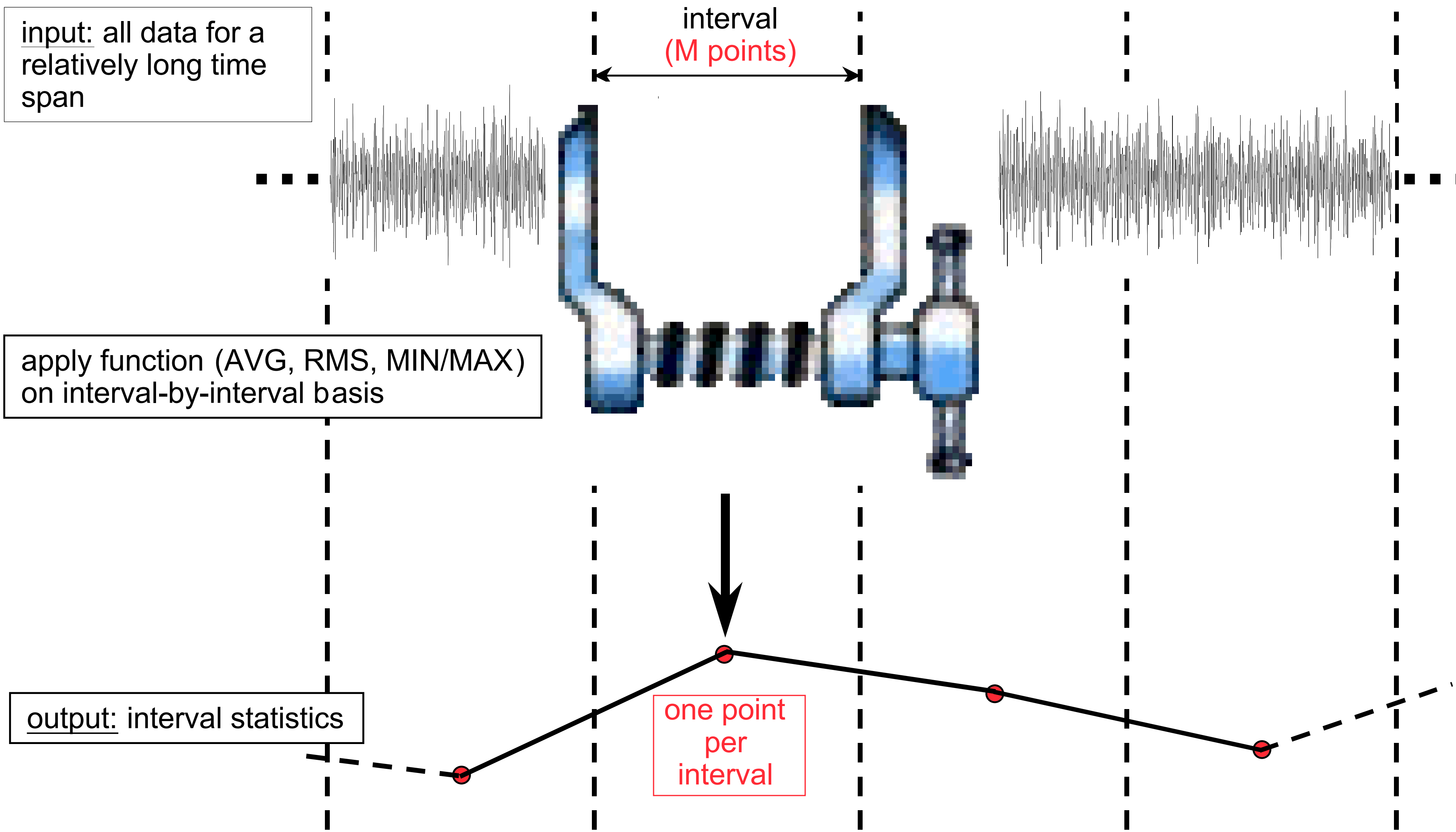
Disadvantages:

- display device (video, printer) constrains resolution for long time spans
- usually not good for qualifying (identifying) acceleration environment

Acceleration vs. Time



Interval Processing



Interval AVG, RMS, MIN/MAX vs. Time

Mathematical Description:

- **AVG**: average (mean) value for each interval

$$x_{AVG}(m) = \frac{1}{M}\sum_{i=1}^{M} x((m-1)M+i); \quad m = 1, 2, \ldots, \left\lfloor \frac{N}{M} \right\rfloor$$

- **RMS**: root-mean-square value for each interval

$$x_{RMS}(m) = \sqrt{\frac{1}{M}\sum_{i=1}^{M} x((m-1)M+i)^2}; \quad m = 1, 2, \ldots, \left\lfloor \frac{N}{M} \right\rfloor$$

- N is number of data points that span the entire interval of interest
- M is the number of data points that span a processing interval
- m is the interval index

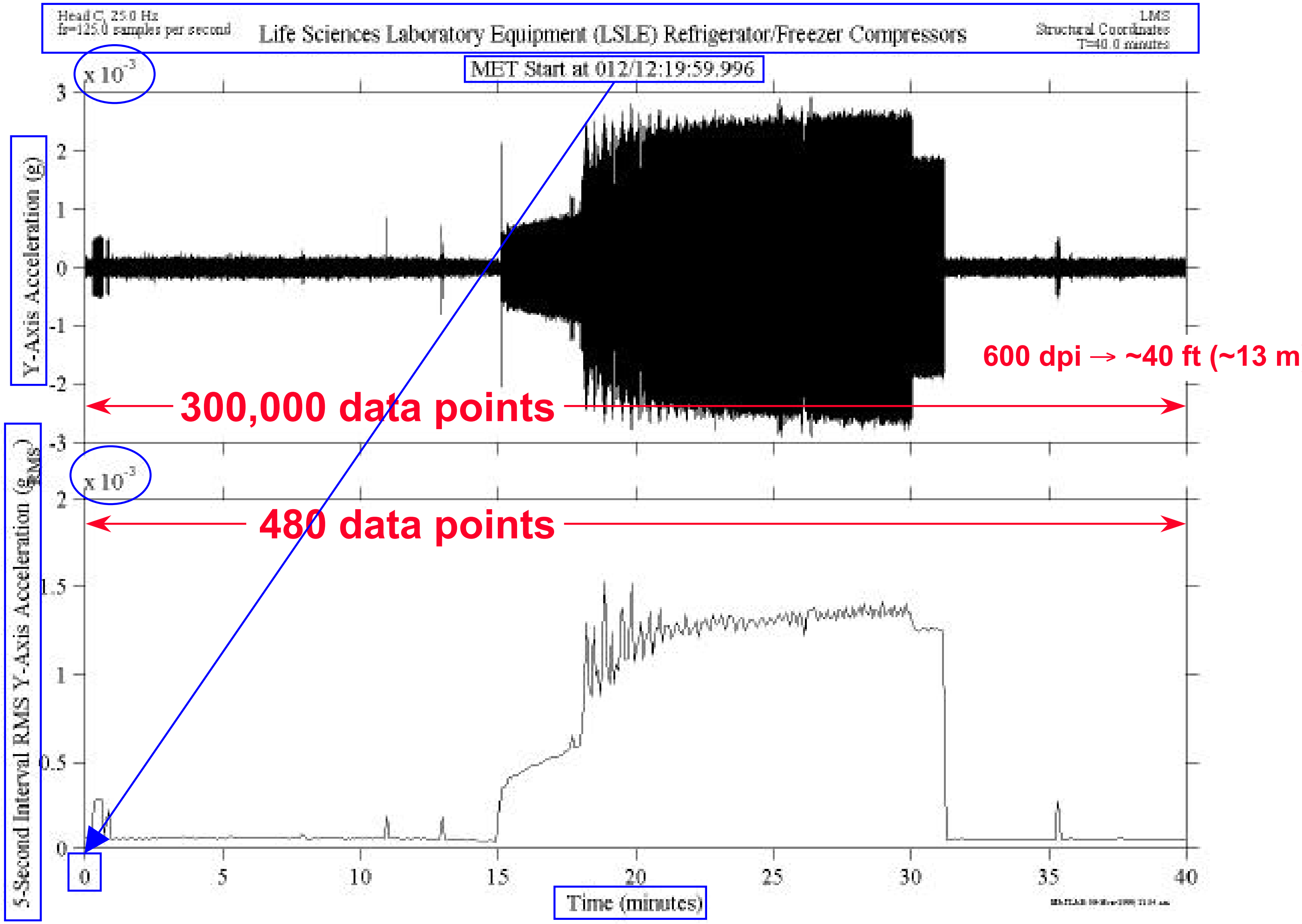
and $\lfloor\ \rfloor$ is the floor function

- **MIN/MAX**: both minimum and maximum values are plotted for each interval – a good display approximation for time histories on output devices with insufficient resolution to display all data in time frame of interest

Advantages: / Disadvantage:

- descriptive statistics……………………………..not-fully-descriptive statistics
- decimation (lossy compression)

Acceleration vs. Time & Interval RMS



Frequency Domain Analysis

Objectives:

- identify and characterize oscillatory acceleration disturbances
- selectively quantify the contribution of various disturbance sources to the overall measured microgravity environment

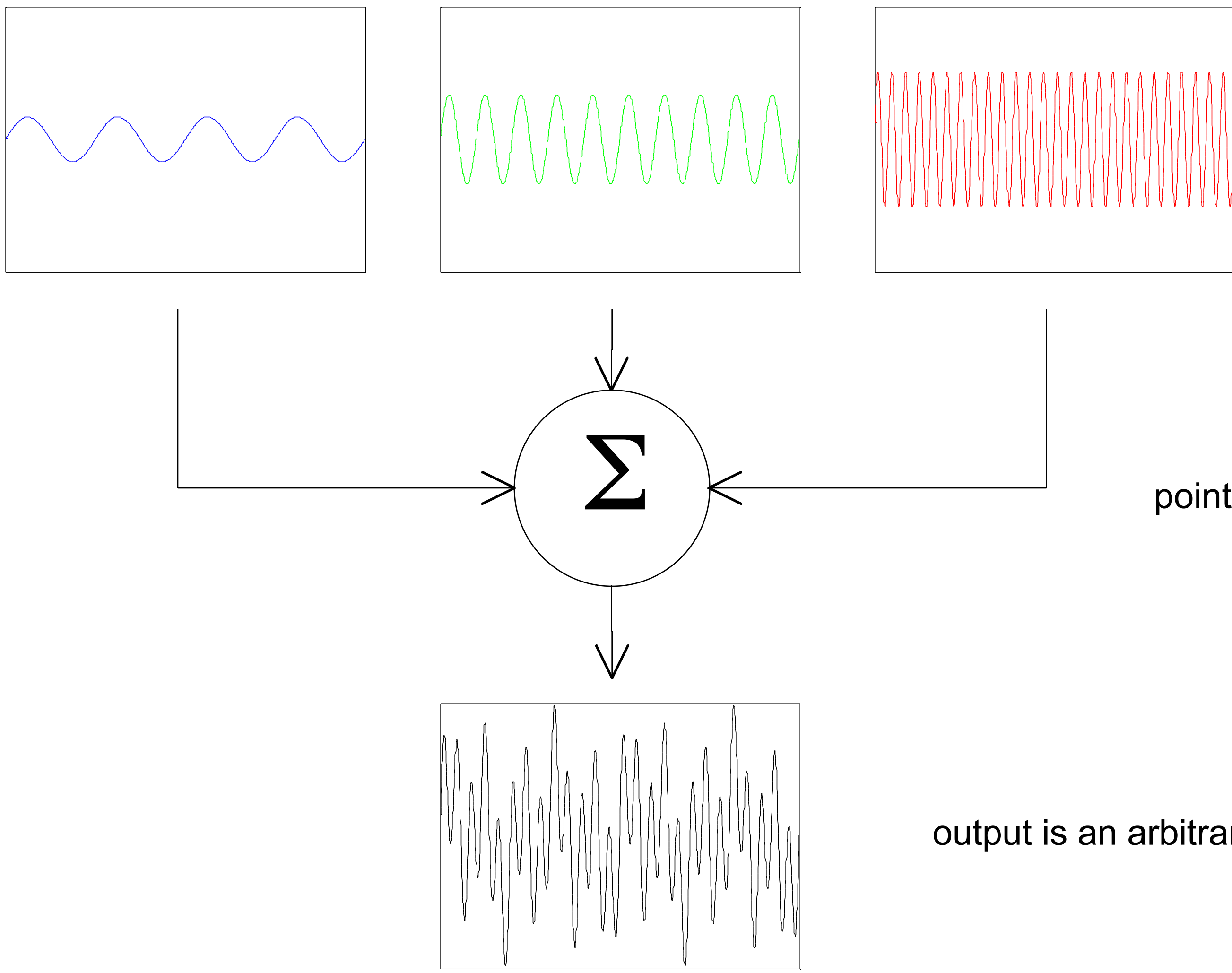
Approaches:

- acceleration power spectral density (PSD) → Parseval's Theorem
- cumulative RMS acceleration vs. frequency
- RMS acceleration vs. one third octave frequency bands
- acceleration spectrogram (PSD vs. *time*)
- principal component spectral analysis (PCSA) vs. frequency

TIME DOMAIN LOOK AT …

Frequency Domain Analysis

Build Arbitrary-Looking Signal

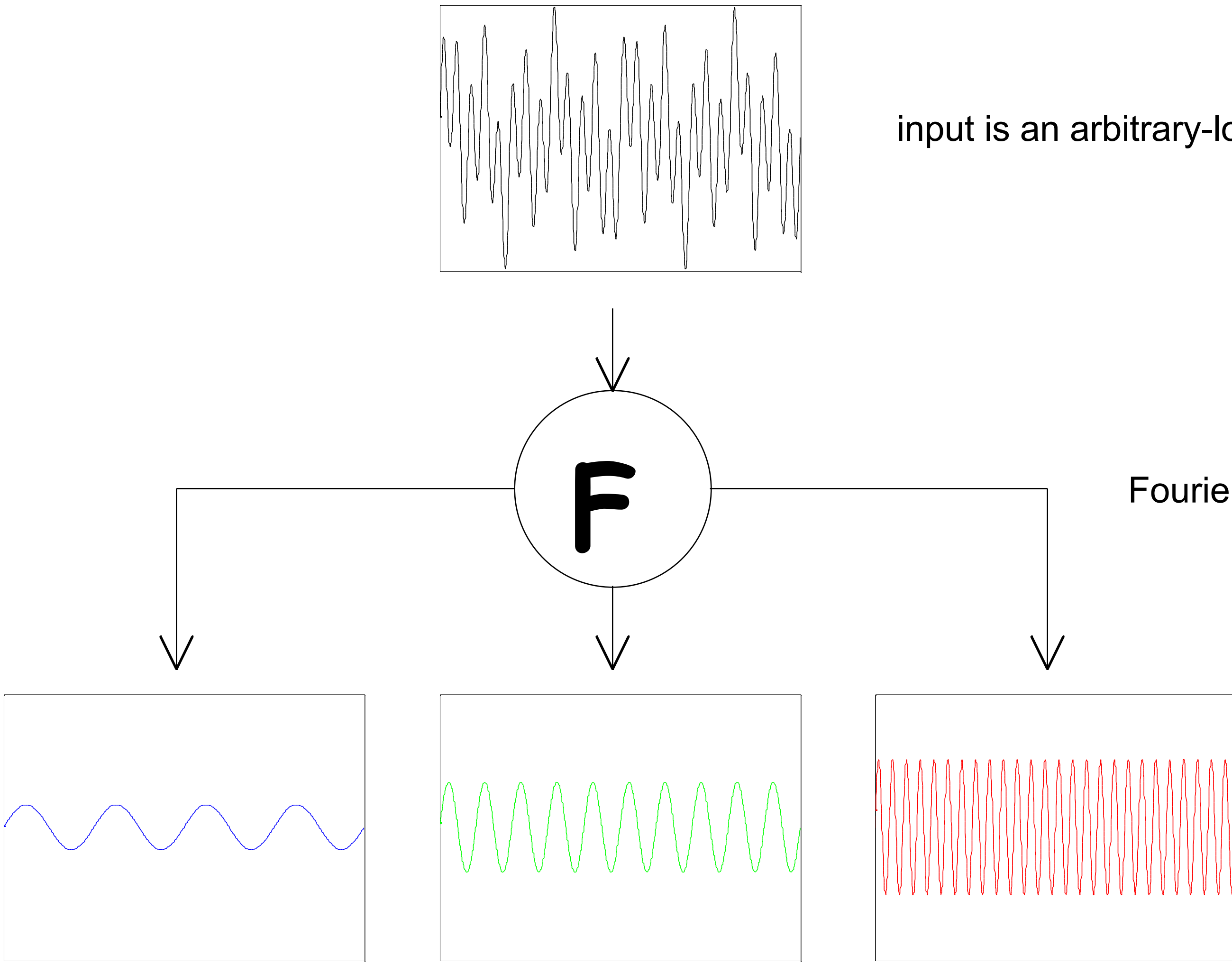


sinusoids with different amplitudes & different frequencies

point-by-point sum

output is an arbitrary-looking signal

Fourier Transform: Graphical Interpretation



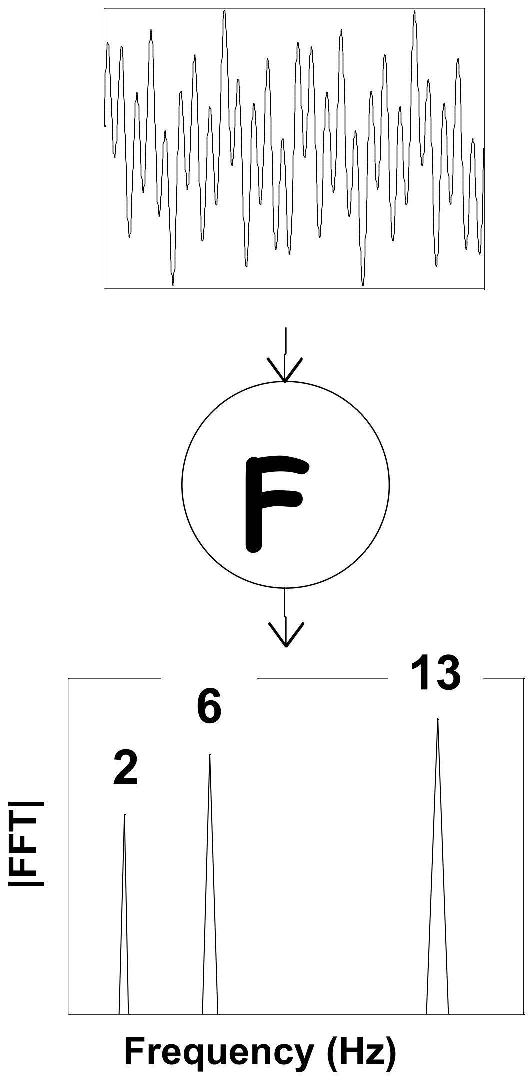
input is an arbitrary-looking signal

Fourier transform

output :

sinusoids with different amplitudes & different frequencies

Fourier Transform: Graphical Description



input is an arbitrary-looking signal

Fourier transform

<u>output :</u>

sinusoids with different amplitudes & different frequencies

Fourier Transform: Mathematical Description

- **What is it?** It's a mathematical transform which resolves a time series into the sum of an average component and a series of sinusoids with different amplitudes and frequencies.
- **Why do we use it?** It serves as a basis from which we derive the power spectral density.
- Mathematically, for continuous time series, the Fourier transform is expressed as follows:

$$X(f) = \int_{-\infty}^{+\infty} x(t) e^{-j2\pi f t} dt \; ; \; j = \sqrt{-1}$$

- For finite-duration, discrete-time signals, we have the discrete Fourier transform (DFT):

$$X(k) = \sum_{n=0}^{N-1} x(n) e^{-j2\pi nk/N} \cancel{\Delta t} \qquad k = 0,1,2, \ldots, (N-1)$$

do not include this factor

$$\boxed{\text{Ä}f = \frac{f_s}{N} = \frac{1}{T}} \quad \boxed{k\Delta f} \qquad \boxed{n\Delta t} \quad \boxed{\text{Ä}t = \frac{1}{f_s}}$$

- N is the number of samples in the time series
- T is the span in seconds of the time series
- f_s is the sample rate in samples/second (Hz)
- Δf is the frequency resolution or spacing between consecutive data points (Hz)

- For a power of two number of points, N, a high-speed algorithm that exploits symmetry is used to compute the DFT. This algorithm is called the fast Fourier transform (FFT).

Power Spectral Density (PSD): Mathematical Description

- **What is it?** It's a function which quantifies the distribution of power in a signal with respect to frequency.
- **Why do we use it?** It is used to identify and quantify vibratory (oscillatory) components of the acceleration environment.
- Mathematically, we calculate the PSD as follows:

$$P(k) = \begin{cases} \dfrac{2\left|X(k)\right|^2}{NUf_s} & [g^2/Hz] \text{ for } k = 1,2,\ldots,(N/2)-1 \\[2ex] \dfrac{\left|X(k)\right|^2}{NUf_s} & [g^2/Hz] \text{ for } k = 0 \text{ and } k = (N/2) \end{cases}$$

$k\Delta f$

DC Nyquist

$$\boxed{U = \frac{1}{N}\sum_{n=0}^{N-1} w(n)^2}$$

- X(k) is the "Δt-less" FFT of x(n)
- N is the number of samples in the time series (power of two)
- f_s is the sample rate (Hz)
- U is window compensation factor
- w(n) is window (weighting) function

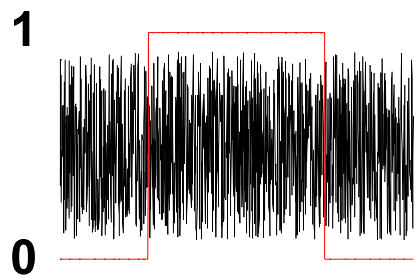
see **Internet References** from earlier slide

- DC is an electrical acronym for direct current that has been generalized to mean average value
- Nyquist frequency (f_N) is the highest resolvable frequency; half the sampling rate ($f_N=f_s/2$)
- Symmetry in the FFT for real-valued time series, x(n), results in one-sided PSDs; only the DC and Nyquist components are unique – that's why no factor of 2 for those in the equation
- Caution: some software package PSD routines scale by some combination of f_s, 2, or N

Windowing to Suppress Spectral Leakage

- **Convolution →spectral leakage**

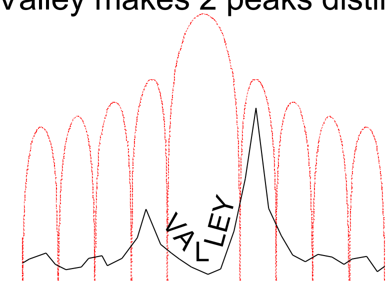
implicit window is called either boxcar, rectangular, square or none



MULTIPLICATION IN TIME DOMAIN

is

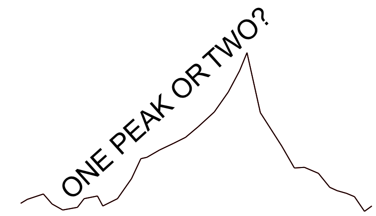
Boxcar has large side lobes
Valley makes 2 peaks distinct



CONVOLUTION IN FREQUENCY DOMAIN

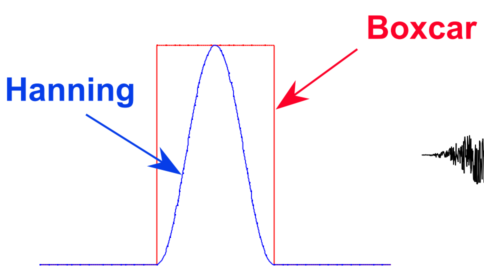
which results in

Leakage into valley obscures distinctness



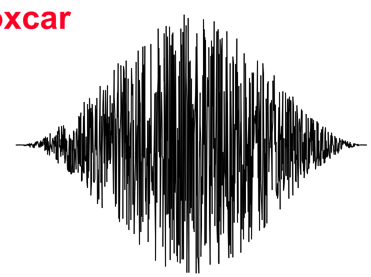
SPECTRAL LEAKAGE IN THE FREQUENCY DOMAIN

- **Taper time series to reduce spectral leakage:**



CHOOSE A WINDOW FUNCTION (TIME DOMAIN)

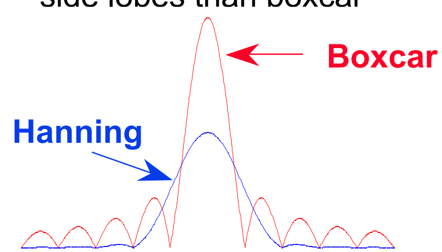
to



TAPER TIME SERIES BY MULTIPLICATION

which results in

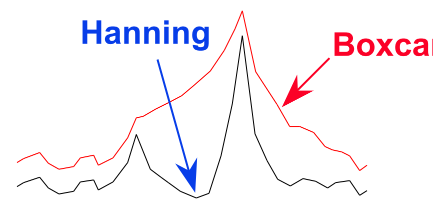
Taper windows have smaller side lobes than boxcar



ATTENUATED SIDELOBES FOR CONVOUTION

that helps

Less leakage into valley improves distinctness



REDUCE SPECTRAL LEAKAGE

- **Window compensation factor** – to account for attenuation of the signal introduced by tapering we apply a compensation factor, U, as shown on the PSD mathematical description slide; U = 1 for the boxcar window

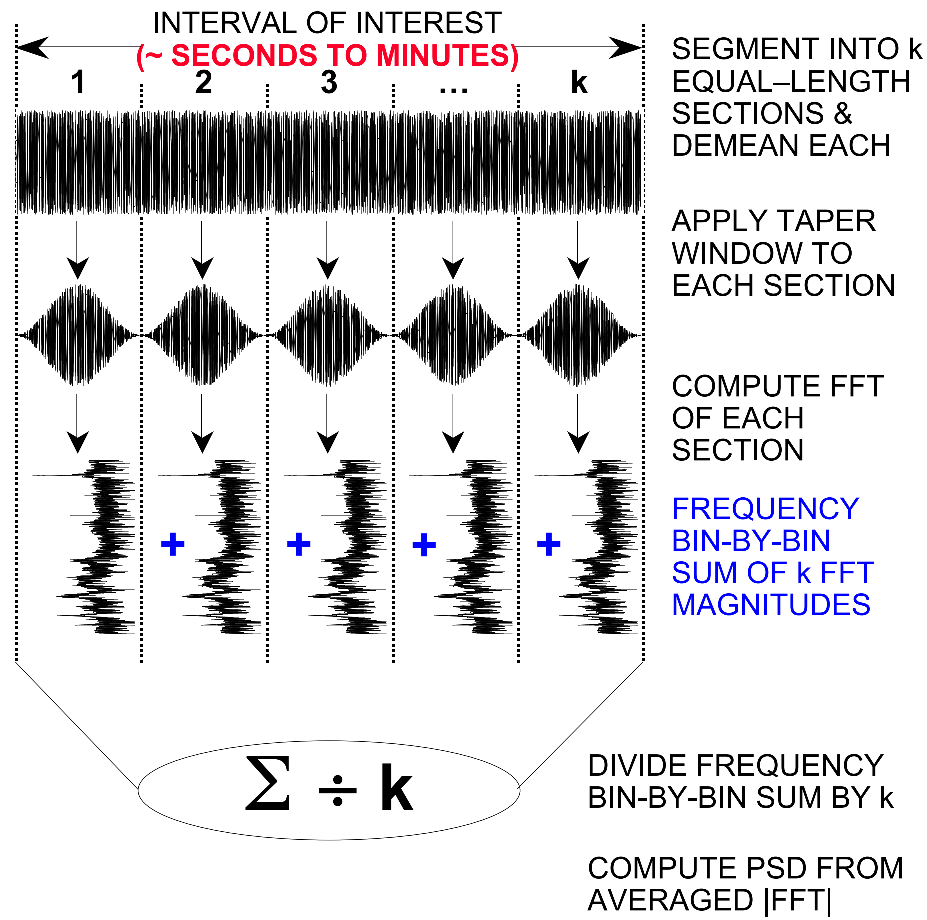
Spectral Averaging

- Assume stationary data
- **Why?** To reduce spectral variance

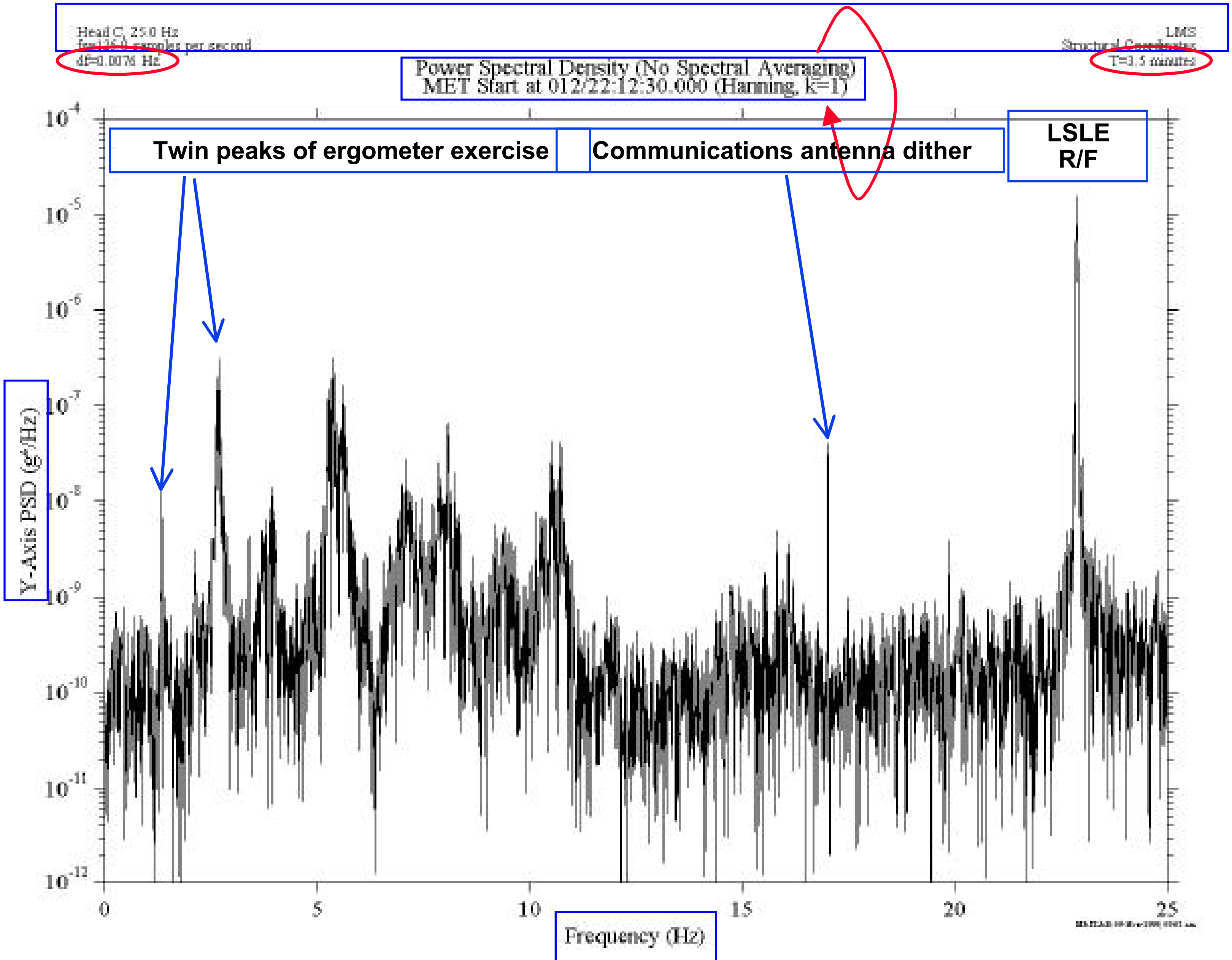
  The averaging in this process causes the variance of the PSD estimate to be reduced by a factor of k.
- **How?** Welch's (periodogram) method
- **Tradeoff:** Degraded frequency resolution

  As the number of averages (or sections, k) increases, the spectral variance decreases, but this comes at the expense of diminished frequency resolution. This stems from the fact that for a given time series, the more sections you have, the fewer the number of points you get in each section.

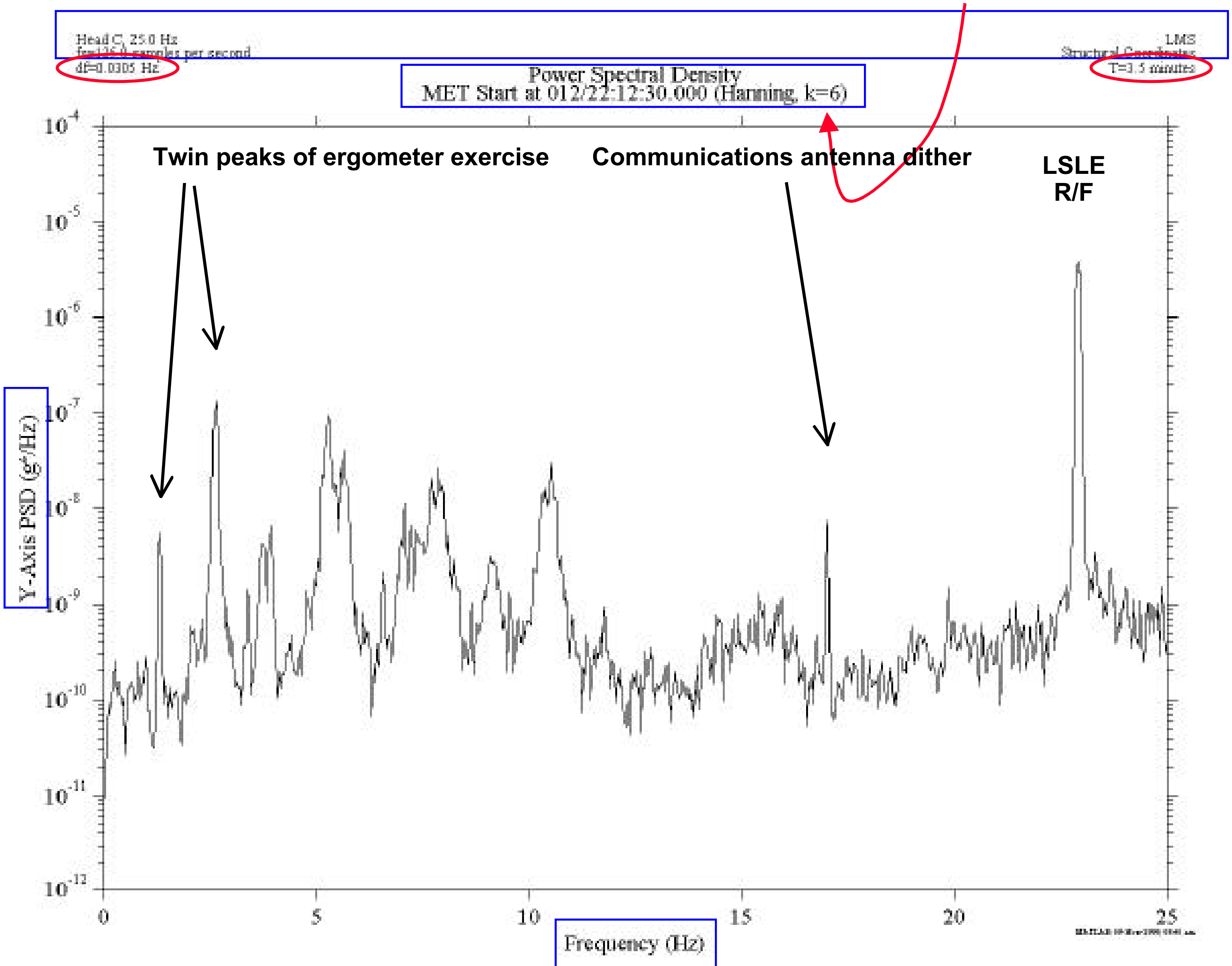
Welch's (Periodogram) Method



Power Spectral Density (no spectral averaging)



Power Spectral Density (with spectral averaging)



Parseval's Theorem

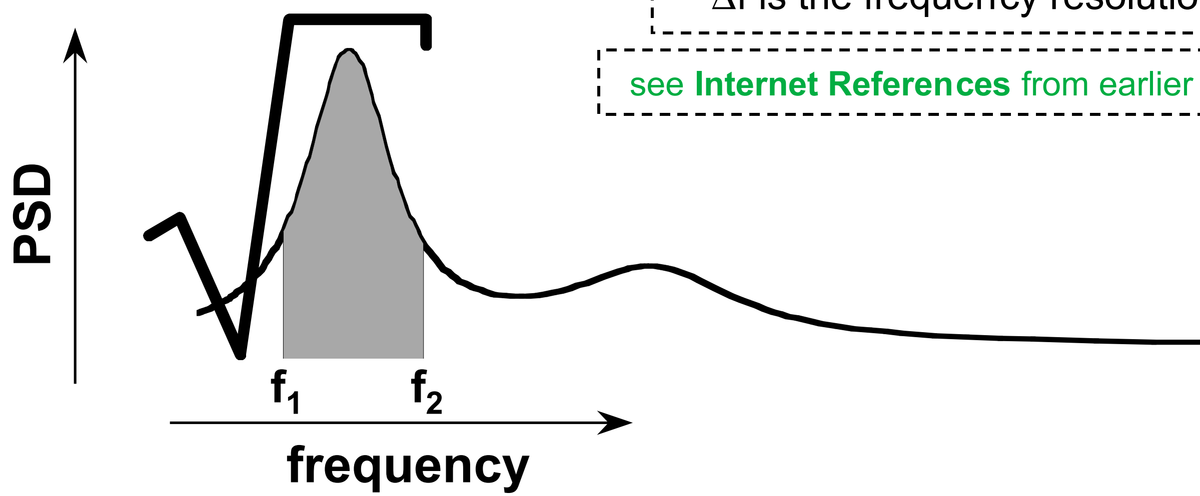
- **What is it?** It's a relation that states an equivalence between the RMS value of a signal computed in the time domain to that computed in the frequency domain.
- **Why do we use it?** It can be used to attribute a fraction of the total power in a signal to a user-specified band of frequencies by appropriately choosing the limits of integration (summation).
- Mathematically, this theorem can be expressed as:

$$\sqrt{\frac{1}{N}\sum_{n=0}^{N-1}|x(n)|^2} = \sqrt{\sum_{k=0}^{N/2}P(k)\Delta f}$$

- $x(n)$ is time series
- N is the number of samples in the time series
- $P(k)$ is the PSD of $x(n)$
- Δf is the frequency resolution

see **Internet References** from earlier slide

$$\left.\mathrm{RMS}\right|_{f_1}^{f_2} =$$



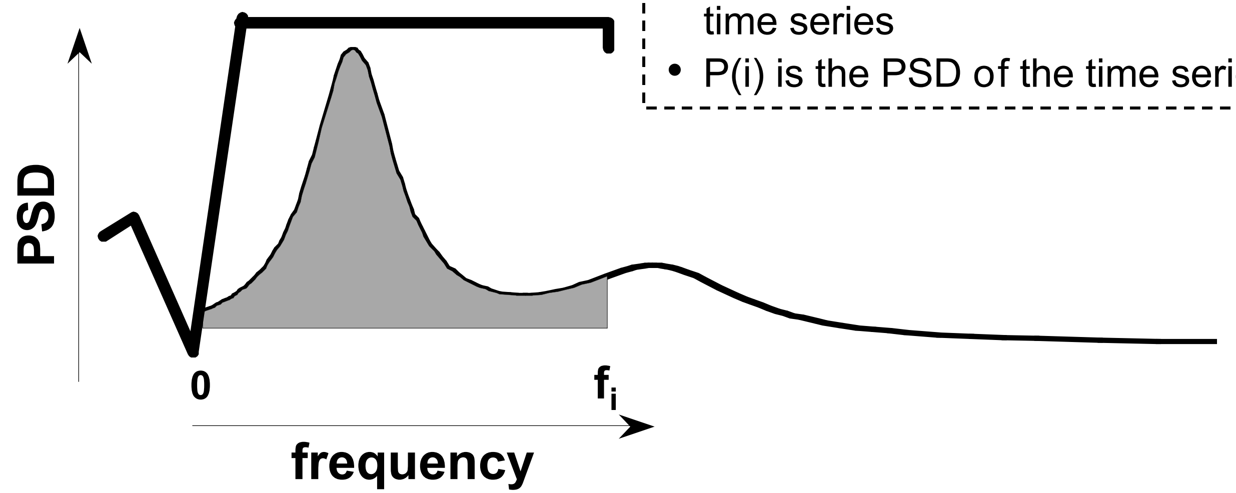
Cumulative RMS vs. Frequency

- **What is it?** It's a plot that quantifies the contributions of spectral components *at and below* a given frequency to the overall RMS acceleration level for the time frame of interest.
- **Why do we use it?** This type of plot highlights, in a quantitative manner, how various portions of the acceleration spectrum contribute to the overall RMS acceleration level.
  - steep slopes indicate strong narrowband disturbances
  - shallow slopes indicate quiet, broadband portions of the spectrum
- Mathematically, we have:

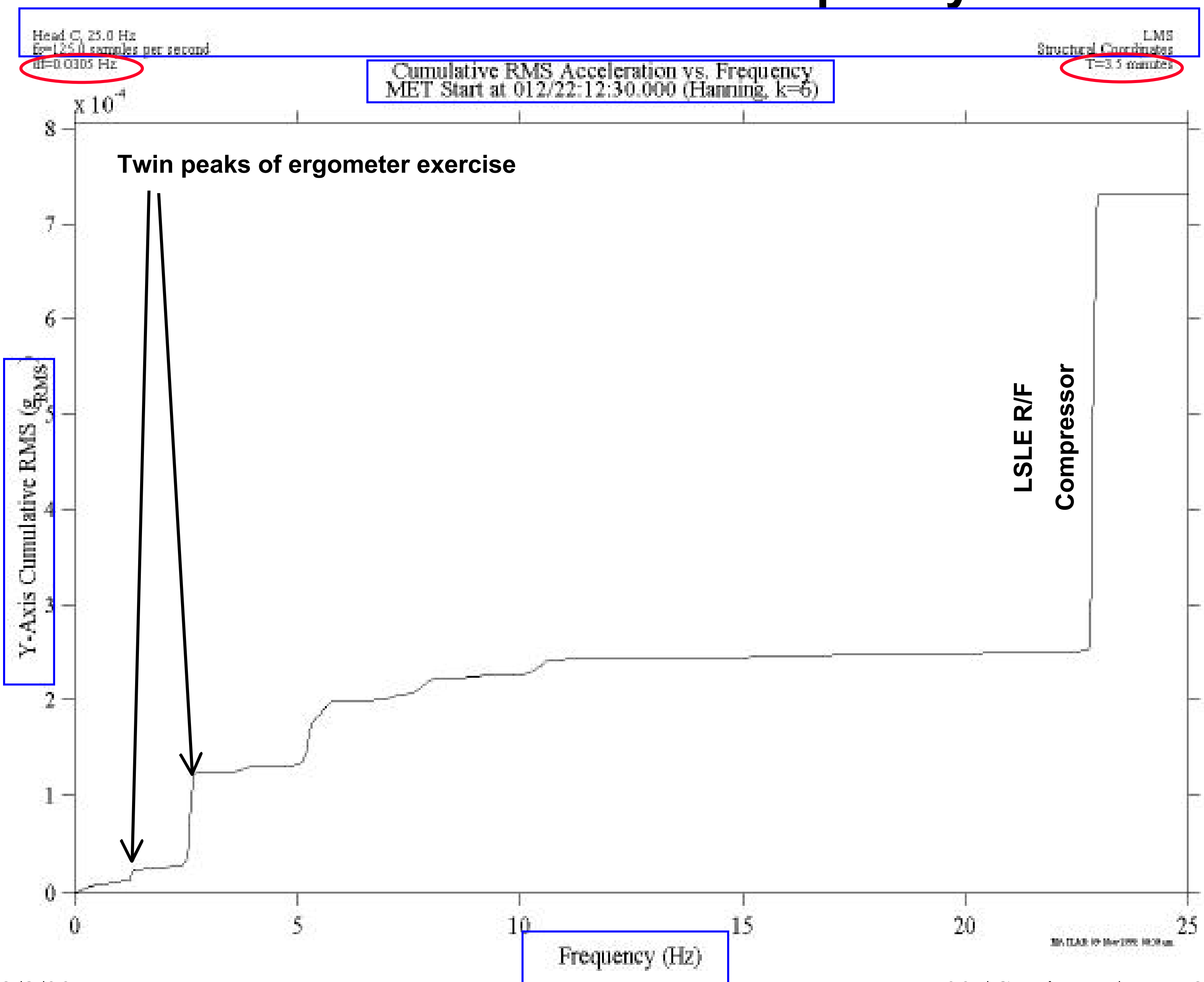
$$a_{RMS}(k) = \sqrt{\sum_{i=0}^{k} P(i)\Delta f} \qquad k = 0,1,2\ldots,(N/2)$$

- Δf is the frequency resolution
- N is the number of samples in the time series
- P(i) is the PSD of the time series

$$RMS\Big|_0^{f_i} =$$



Cumulative RMS vs. Frequency



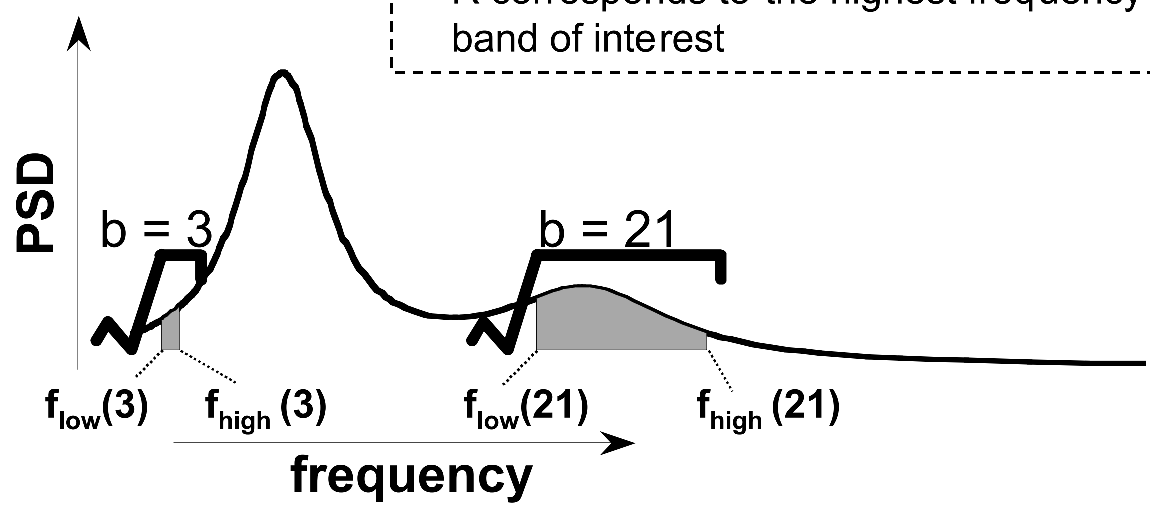
RMS vs. One Third Octave Frequency Bands

- **What is it?** It's a plot that quantifies the spectral content in proportional bandwidth frequency bands for a given time interval of interest.
- **Why do we use it?** The International Space Station vibratory limit requirements are defined in terms of the RMS acceleration level for each of 31 one third octave bands with the time interval specified as 100 seconds.
- Mathematically, we have:

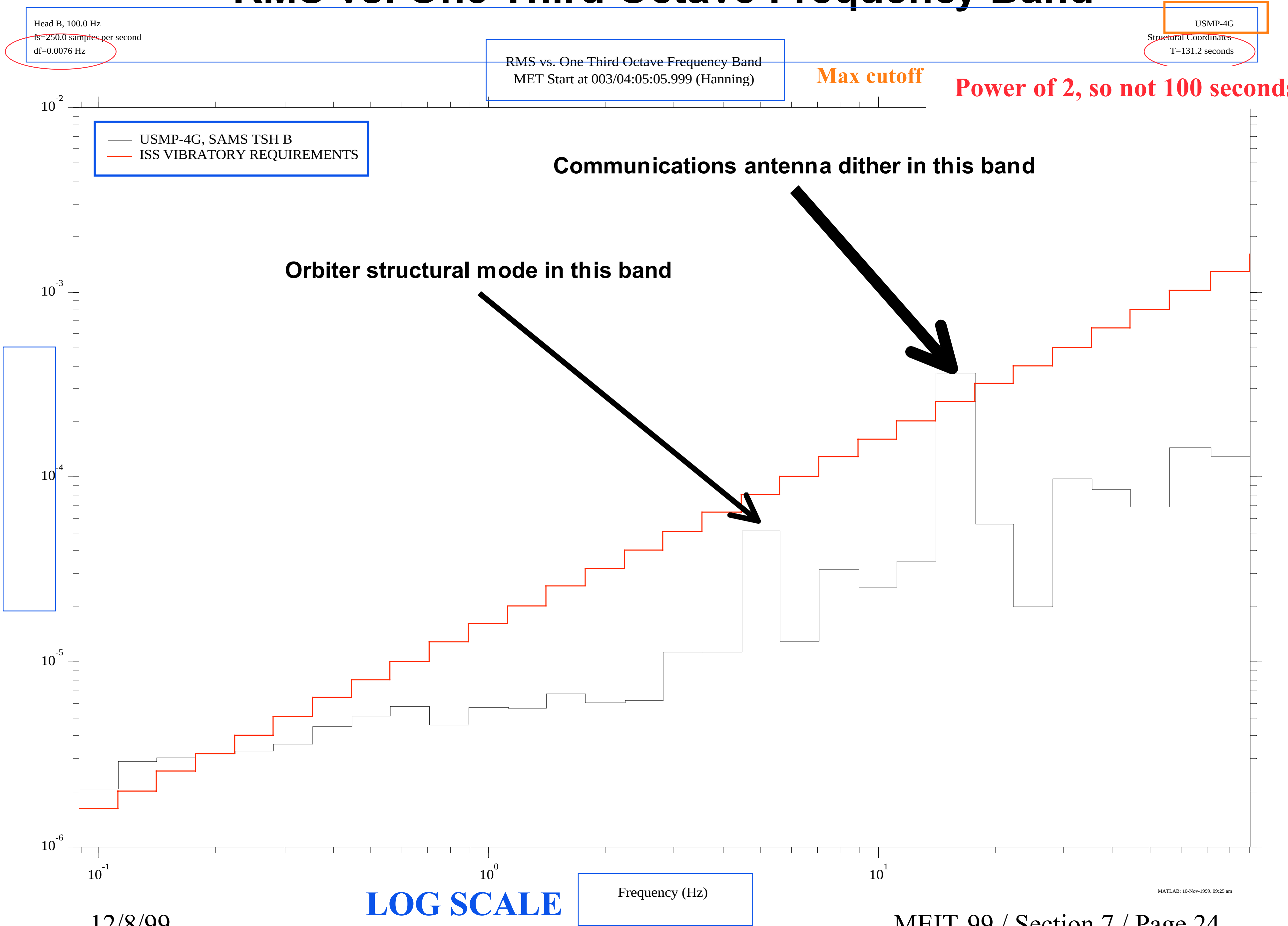
$$a_{RMS}(b) = \sqrt{\sum_{i=f_{low}(b)}^{f_{high}(b)} P(i)\Delta f} \qquad b = 1,2\ldots,R$$

- $f_{low}(b)$ and $f_{high}(b)$ are frequency indices for the b^{th} one third octave band
- $P(i)$ is the PSD of the time series
- Δf is the frequency resolution
- R corresponds to the highest frequency band of interest

$$RMS_b \Big|_{f_{low}(b)}^{f_{high}(b)} =$$



RMS vs. One Third Octave Frequency Band

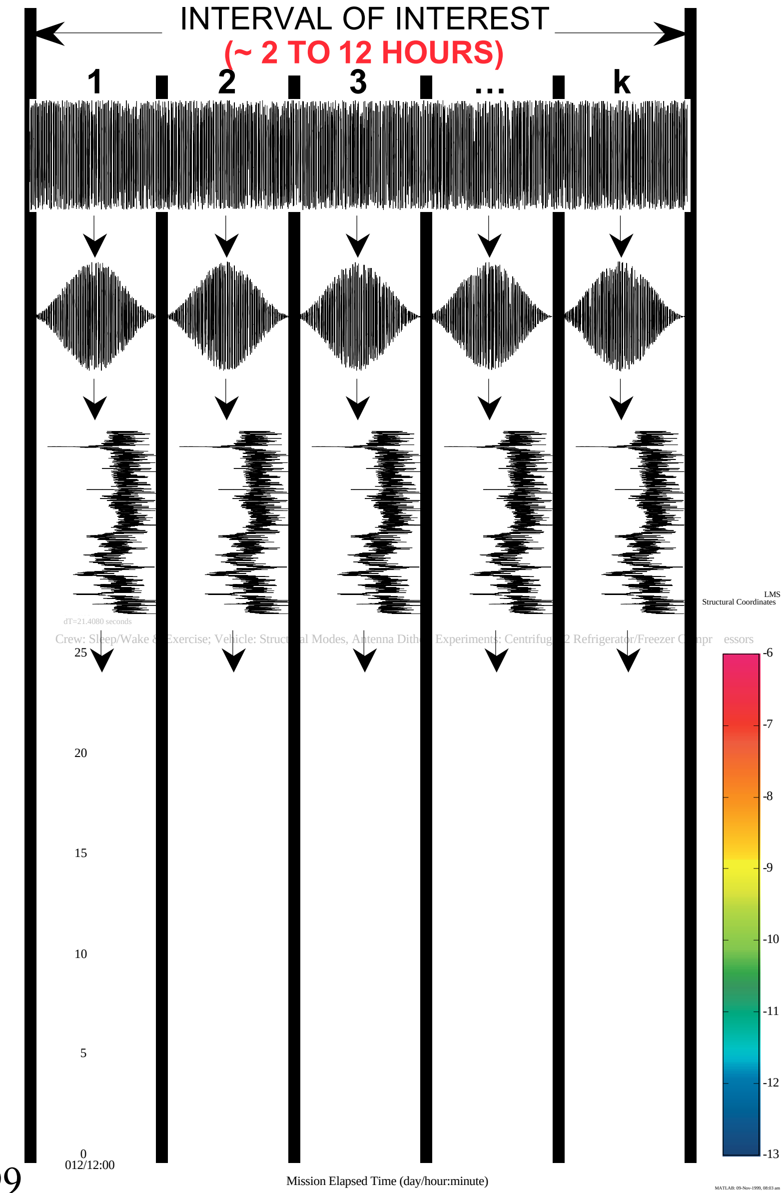


Spectrogram

- **What is it?** It's a three-dimensional plot that shows PSD magnitude (represented by color) versus frequency versus time.
- **Why do we use it?**
  - It is a powerful qualitative tool for characterizing long periods of data
  - Identification and characterization of boundaries and structure in the data
    - Determine start/stop time of an activity within temporal resolution, dT (dT is not $\Delta t \leftarrow$ overlap)
    - Track frequency characteristics of various activities within frequency resolution, Δf
- **Things you should NOT do with a spectrogram:**
  - Quantify disturbances in an absolute sense. The cumulative RMS or one-third octave versus frequency plots are better suited for this objective.
  - Rely entirely on it to check for the presence of a disturbance which is either known or expected to be relatively weak. A PSD with appropriate spectral averaging works better for this.

Frequency Domain Analysis

How to Build a Spectrogram



1. SEGMENT INTO k EQUAL–LENGTH SECTIONS AND DEMEAN EACH SECTION

2. APPLY TAPER WINDOW TO EACH SECTION

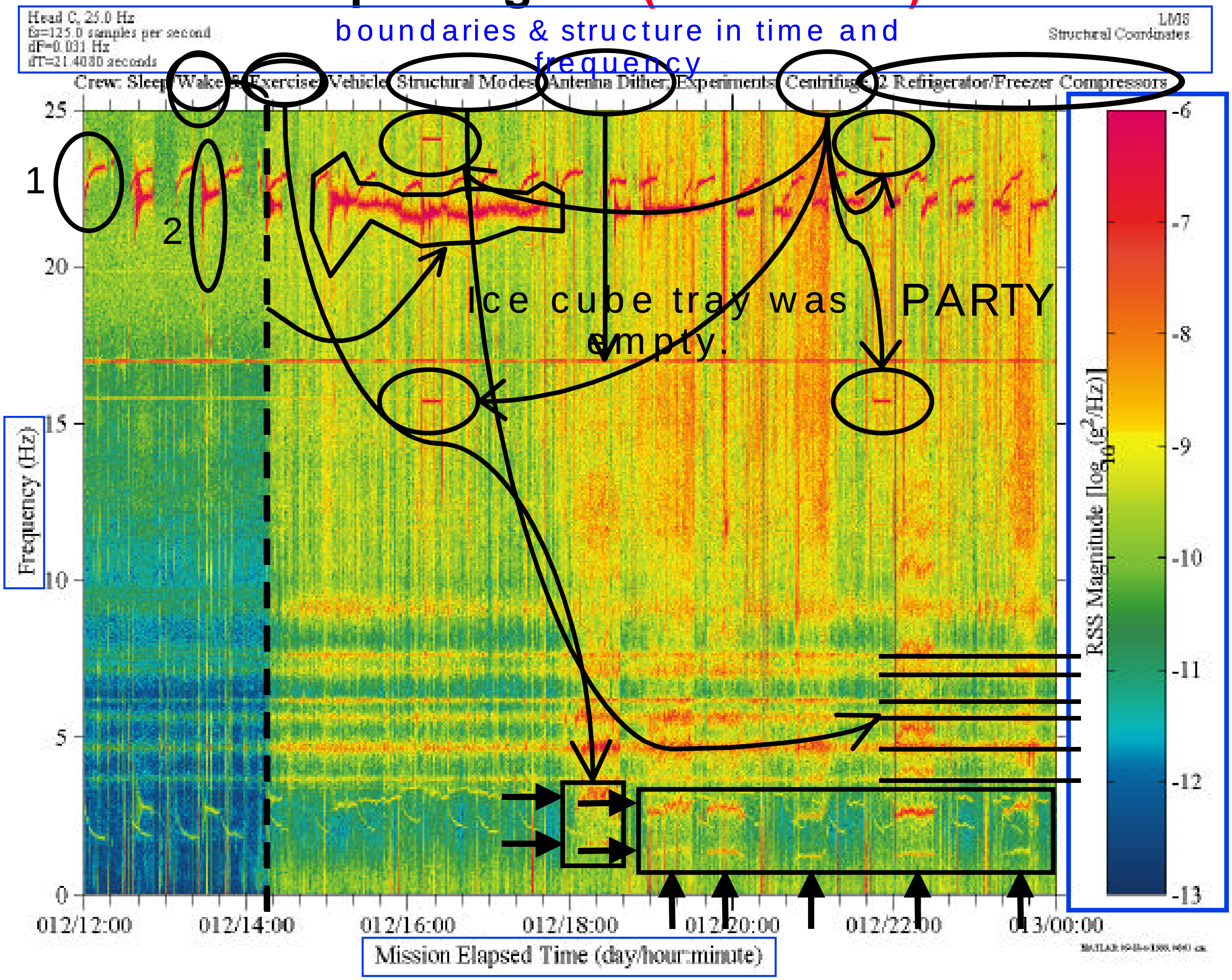
3. COMPUTE PSD OF EACH SECTION

4. CALCULATE LOG_{10} OF |PSDs| AND MAP NUMERIC VALUES TO COLORS SUCH THAT THE BLUE (BOTTOM) PART OF THE COLOR MAP REPRESENTS SMALLER VALUES THAN THOSE TOWARD THE RED (TOP) PART

5. DISPLAY EACH OF THE k PSD SECTIONS AS A VERTICAL STRIP OF THE SPECTROGRAM (LIKE WALLPAPERING), SUCH THAT TIME INCREASES FROM LEFT TO RIGHT AND FREQUENCY INCREASES FROM BOTTOM TO TOP

Note: The width of each strip is the temporal resolution and the height of each distinct color patch is the frequency resolution.

Spectrogram (12 HOURS)



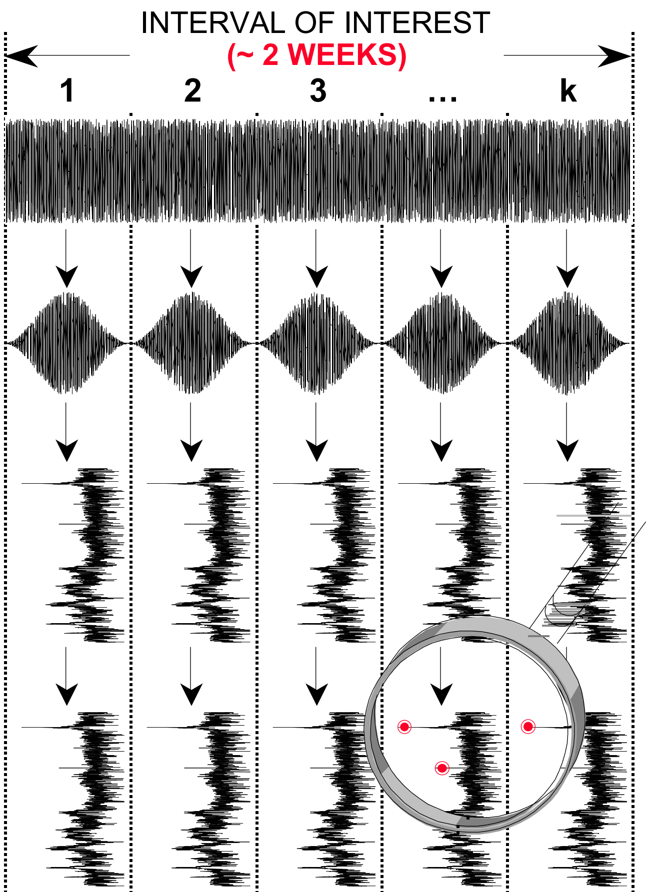
Principal Component Spectral Analysis (PCSA)

- **What is it?** A frequency domain analysis technique that employs a peak detection algorithm to accumulate PSD magnitude and frequency values of dominant or persistent spectral contributors and display them in the form of a 2-D histogram.

- **Why do we use it?** To examine the spectral characteristics of a long period of data.
  - serves to *summarize* magnitude and frequency variations of key spectral contributors
  - better frequency and PSD magnitude resolution relative to a spectrogram

- **Tradeoff:** Poor temporal resolution – no direct way of correlating with time

PCSA Procedure



1. SEGMENT INTO k EQUAL–LENGTH SECTIONS AND DEMEAN EACH SECTION

2. APPLY TAPER WINDOW TO EACH SECTION

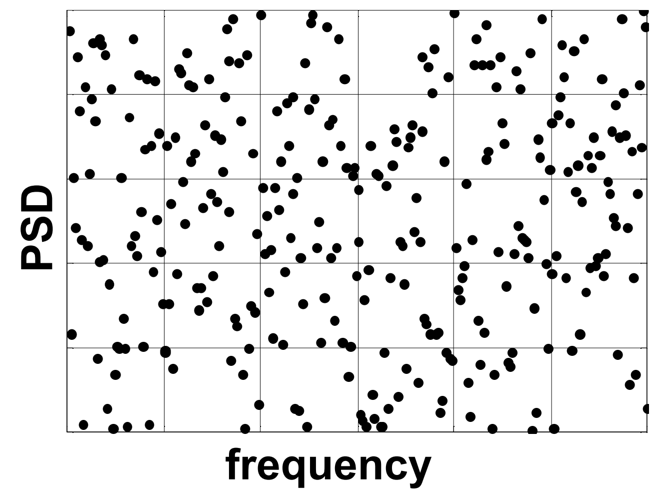
3. COMPUTE PSD OF EACH SECTION

4. PASS EACH PSD THROUGH A PEAK DETECTOR (FIND ALL THE PEAKS THAT ARE LOCAL MAXIMA AND AT LEAST AS HIGH AS ANY OTHER POINT WITHIN A USER-DEFINED NEIGHBORHOOD OF FREQUENCY BINS) TO YIELD PEAK PSD MAGNITUDES VERSUS FREQUENCY

5. STORE THESE PEAKS AND CORRESPONDING FREQUENCIES AS INTERMEDIATE RESULTS

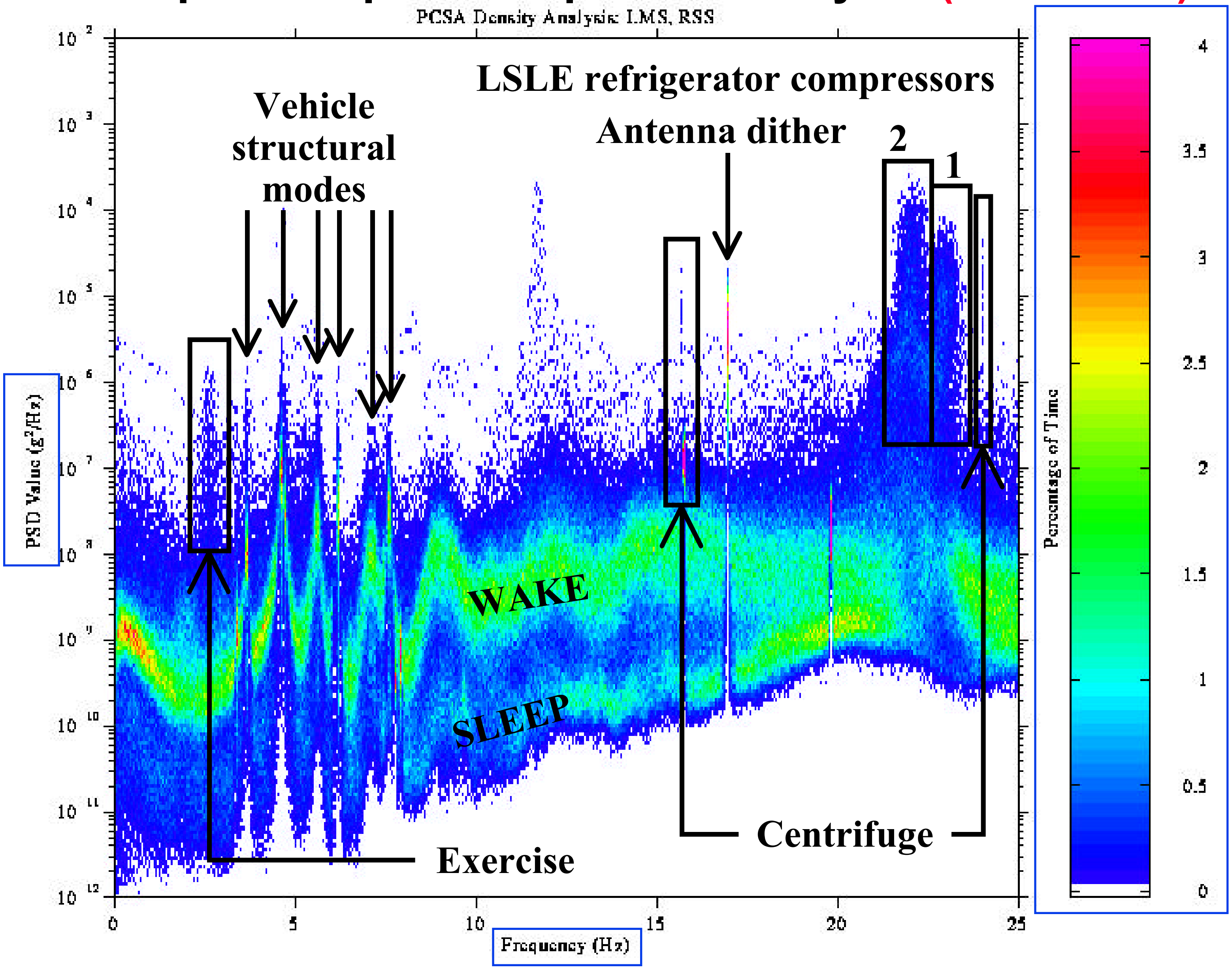
6. PLOT THE ACCUMULATED PSD PEAKS VERSUS FREQUENCY AS A TWO-DIMENSIONAL HISTOGRAM COLOR DENSITY PLOT

Magnitude-Frequency Bins for 2-D Histogram



NOTE: DESIRED FREQUENCY RESOLUTION AND POWER-OF-TWO CONSIDERATIONS WILL DETERMINE THE INTERVAL USED IN STEP 1; HOWEVER, THIS PROCEDURE ALLOWS FOR ARBITRARILY LARGE TIME SPANS TO BE CONSIDERED

Principal Component Spectral Analysis (15+ DAYS)



Time Domain Summary Table

<table>
<tr><th>DISPLAY</th><th>NOTES</th></tr>
<tr><td>Acceleration vs. Time</td><td>• most precise accounting of measured data with respect to time
• display device constrains resolution for long time spans</td></tr>
<tr><td>Interval Minimum/Maximum Acceleration vs. Time</td><td>• displays upper and lower bounds of peak-to-peak excursions
• good display approximation for time histories on output devices with resolution insufficient to display all data in time frame of interest</td></tr>
<tr><td>Interval Average Acceleration vs. Time</td><td rowspan="2">• descriptive statistics
• not fully descriptive (lossy compression)</td></tr>
<tr><td>Interval Root-Mean-Square (RMS) Acceleration vs. Time</td></tr>
</table>

Frequency Domain Summary Table

DISPLAY	NOTES
Power Spectral Density (PSD) vs. Frequency	• quantifies distribution of power with respect to frequency • windowing (tapering) to suppress spectral leakage • spectral averaging to reduce spectral variance (degraded Δf)
Cumulative RMS Acceleration vs. Frequency	• quantifies RMS contribution at and below a given frequency • quantitatively highlights key spectral contributors
RMS Acceleration vs. One Third Octave Frequency Bands	• quantify RMS contribution over proportional frequency bands • compare measured data to ISS vibratory requirements
Spectrogram (PSD vs. Frequency vs. Time)	• displays power spectral density variations with time • good *qualitative* tool for characterizing long periods • identify structure and boundaries in time and frequency
Principal Component Spectral Analysis (PCSA)	• summarize magnitude and frequency excursions for key spectral contributors over a relatively long period of time • results typically have finer frequency resolution and high PSD magnitude resolution relative to a spectrogram at the expense of poor temporal resolution